

# Overview January 2013

## Structure of Doctoral-Research at the Welfenlab

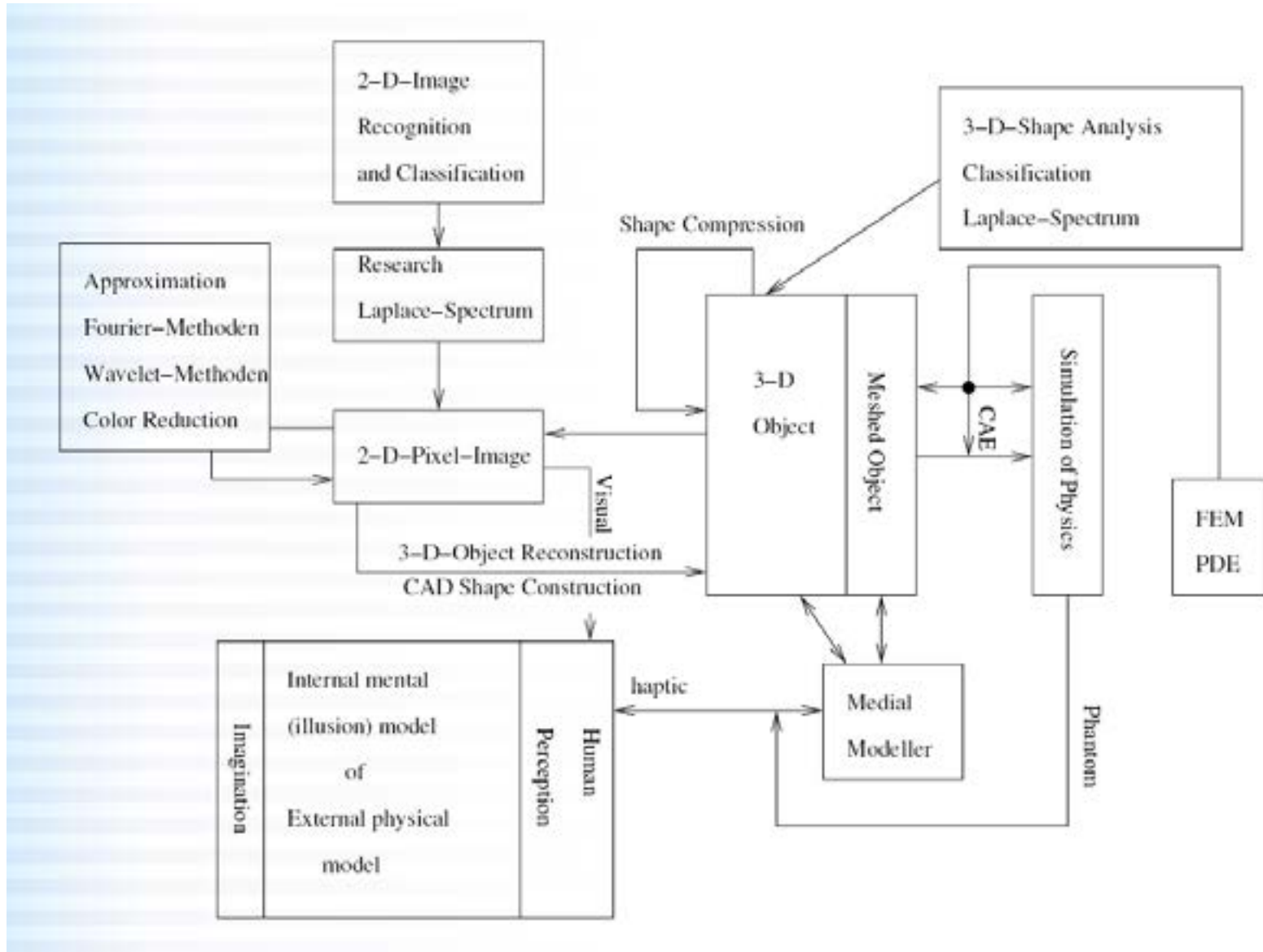


F.-E. Wolter

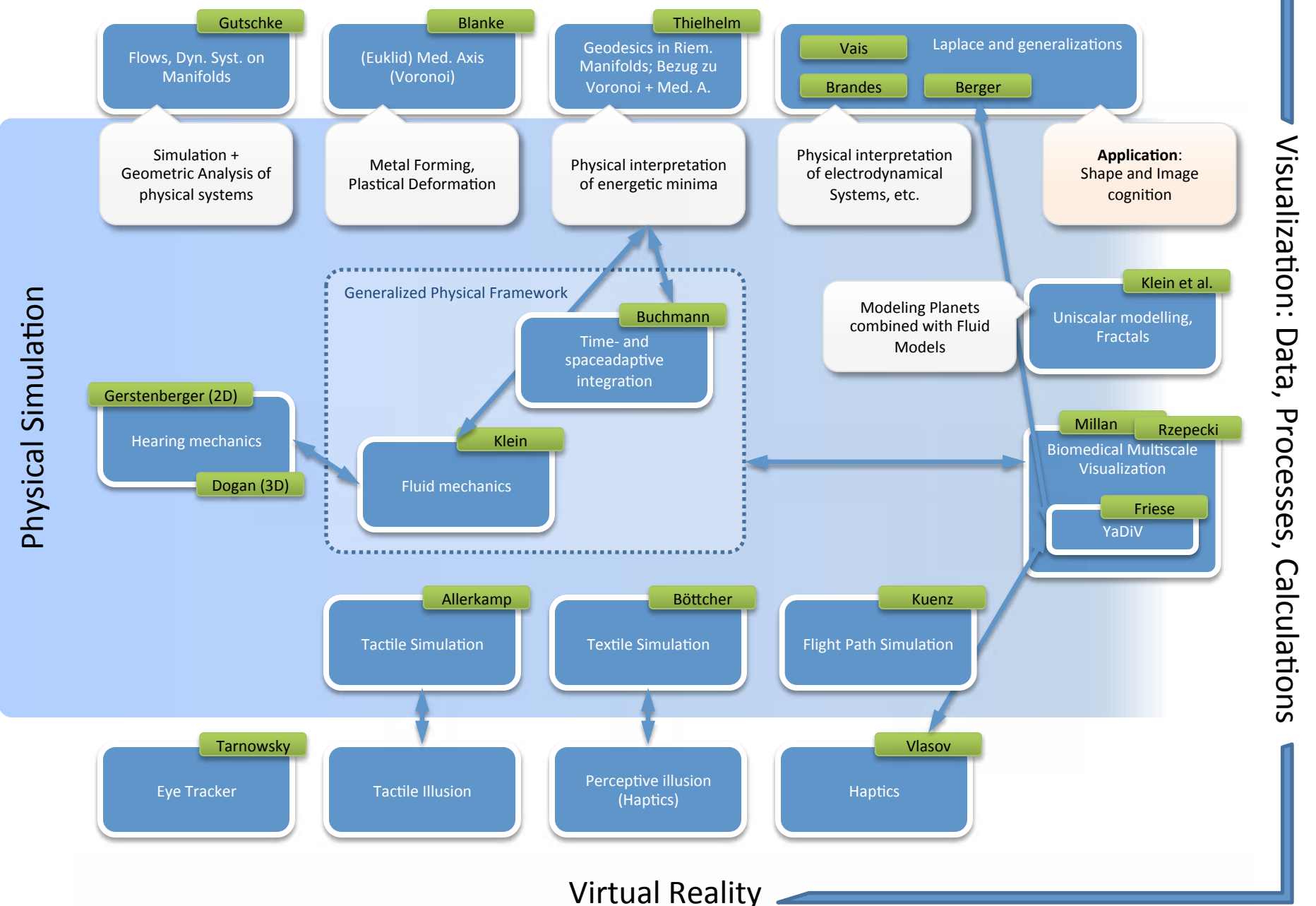
Welfen Laboratory

Institute of Man-Machine-Communication  
Leibniz University of Hannover

# RELATION OF CONCEPTS (AROUND 2003)



# Computational Geometry + Topology



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# 0. PREFACE: DYNAMICAL SYSTEMS



# DYNAMICAL SYSTEMS

**General form:** e.g.

$$x''(t) = M^{-1} \cdot F(x(t), x'(t))$$

e.g.: used in Haptex simulations cf. slide 23

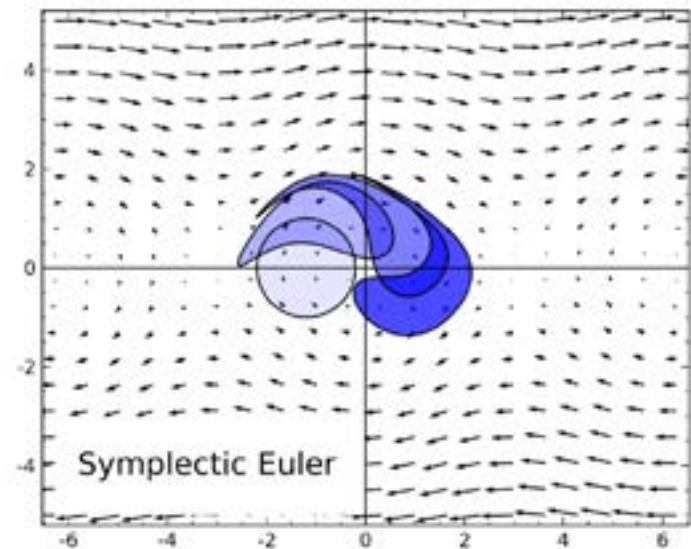
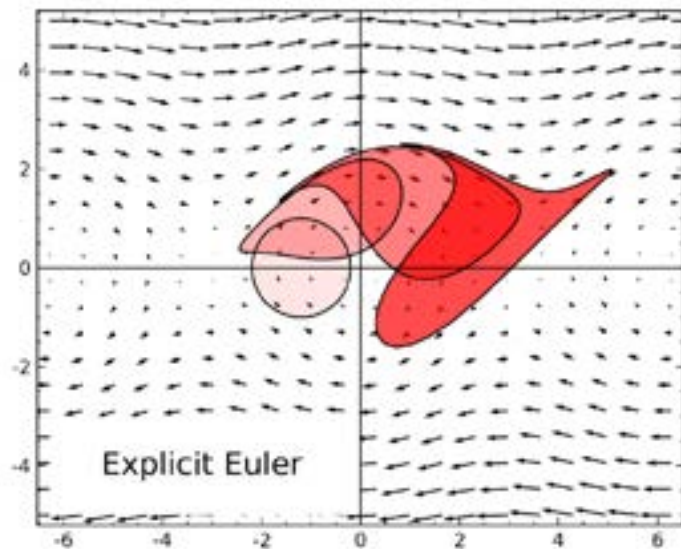
# DYNAMICAL SYSTEMS

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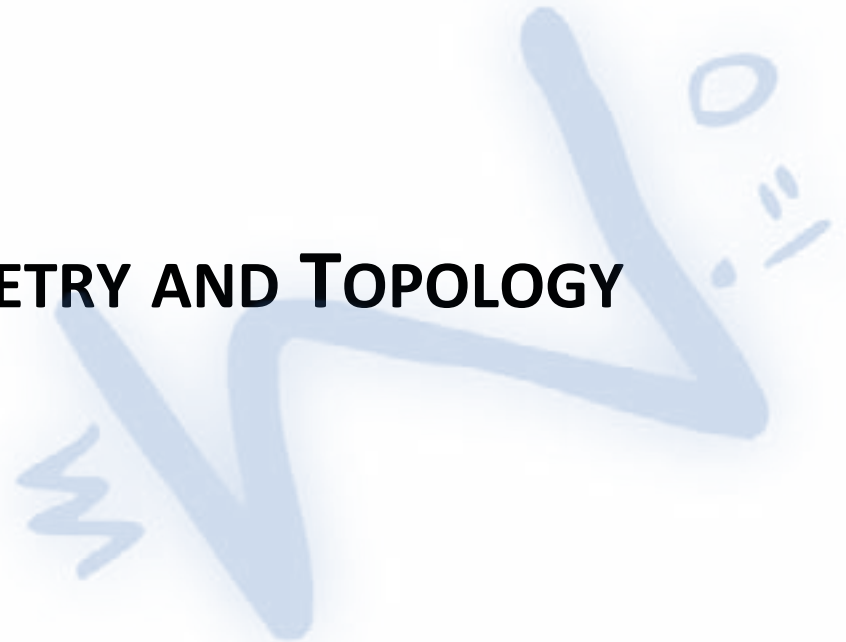
## Example: Pendulum

- Energy (area) should be conserved
- Result quality depends on chosen integration method



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# 1. COMPUTATIONAL GEOMETRY AND TOPOLOGY



# DYNAMICAL SYSTEMS ON SUBMANIFOLDS IN EUCLIDEAN SPACE

## Problem Statement

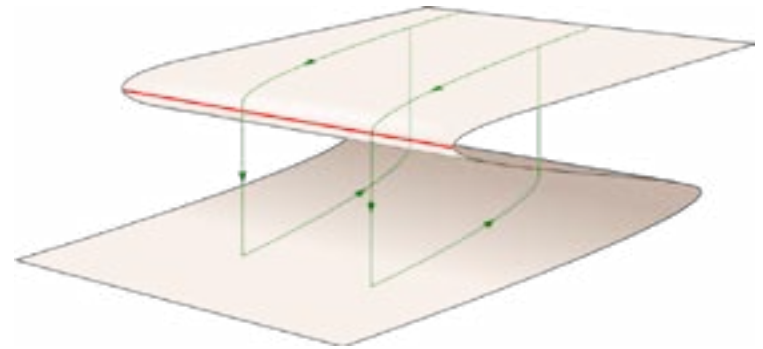
- Study Solutions (Flow Map)  $\varphi(x, t)$  of  $y' = G(x, t)$   
 $t \mapsto \varphi(x, t)$  with  $\frac{d}{dt} \varphi(x, t) = G(\varphi(x, t), t)$
- Analyze dynamical system with orbits on submanifold  $S$
- $S$  defined implicitly by  $F(x) = c$
- Following a point controlled by this system
- Point moves slow while on  $S$  and fast while not on  $S$ .



# DYNAMICAL SYSTEMS ON SUBMANIFOLDS IN EUCLIDEAN SPACE

## Applications

- Analyzing Physical Systems,
  - electrical circuit system
  - Mechanical and biological systems
- Helpful studying periodic orbits



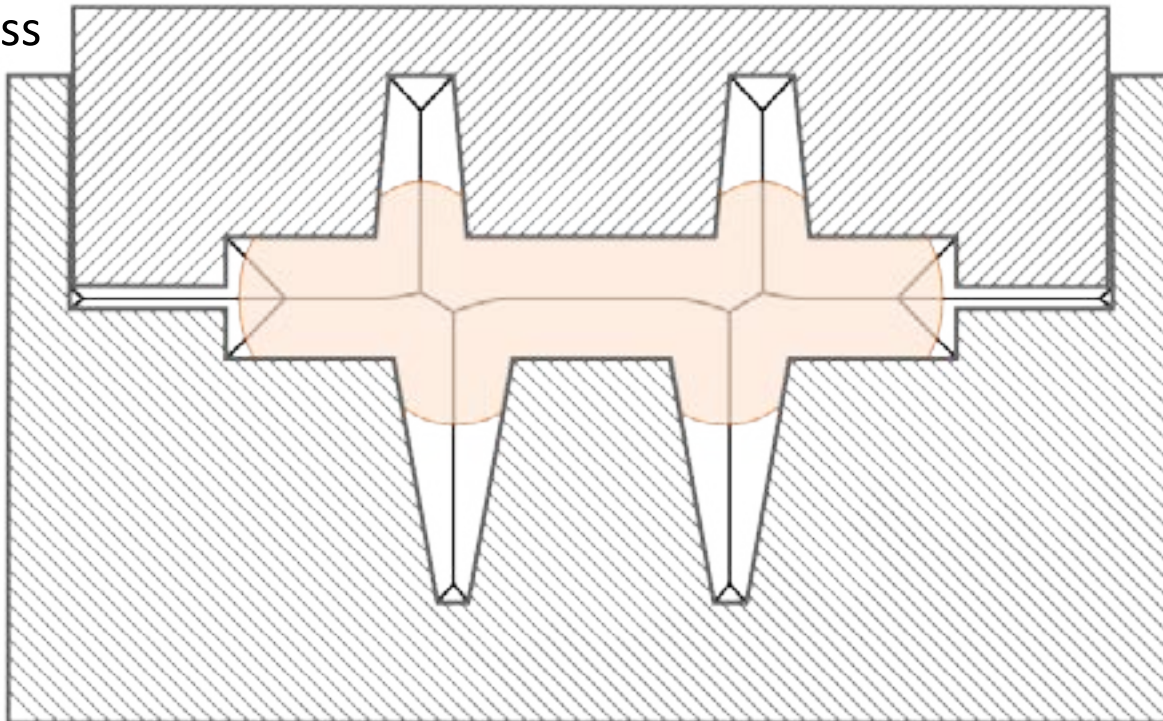
# SPECIAL PLASTIC DEFORMATIONS & MEDIAL AXIS

## Problem Statement

- Compute the inverse  $\varphi^{-1}$  of a computable Flow map  $\varphi(x, t)$
- Interesting in Metal Forming process

- This problem turns out to be very difficult, see:
  - “Current theoretical approaches to collective behavior of dislocations”, G. Ananthakrishna
  - “Plastic Deformations and Dynamical Systems”, Kubin, L.P., Fressengeas, C., Ananthakrishna, G. (2002)

*Behrens, B.A., Blanke, P., Hadifi, T., Wolter, F.-E., Santangelo, A.: Fast 3D inverse simulation of hot forging processes via Medial Axis Transformation: An approach for preform estimation in hot die forging. Prod. Eng. 7, 409–416 (2013).*



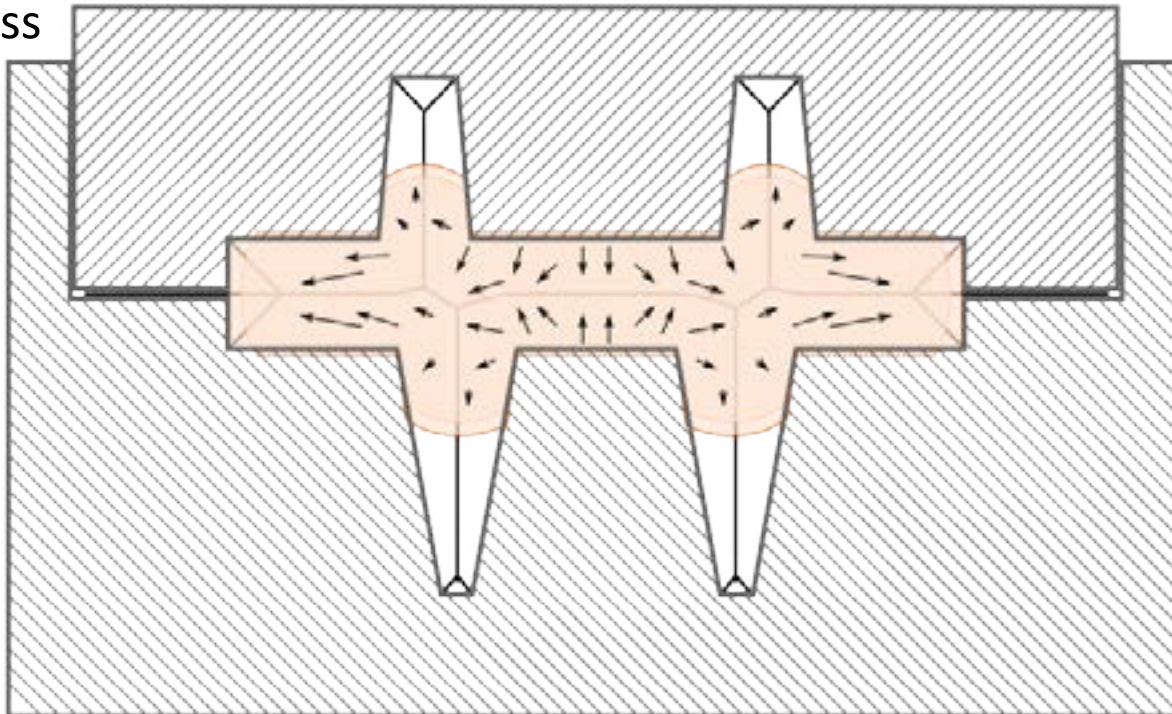
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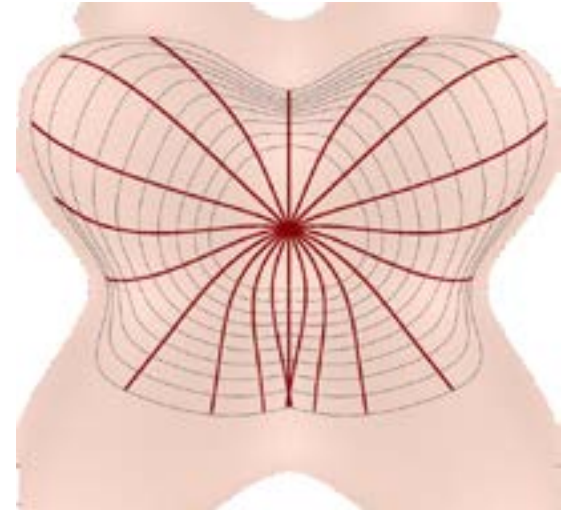
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# GEODESICS IN RIEMANNIAN MANIFOLDS

- Described by special dynamical systems defined by the geometry of the manifold
- Differential equation:  $y' = \Gamma(y, t)$
- Special flow maps: Offset/Exponential maps  $O(y, t)$  offset geometries
- More specialized notation:  
 $\exp(x, t)$ , inverse:  $\exp^{-1}(\exp(x, t))$



- 
- Study Focal Sets, i.e. where  $D\exp$  has not maximal rank
  - Study Flow Map, its inverse and singularities
    - Use it to analyze the distance geometry of a Riemannian Manifold

# GEODESICS IN RIEMANNIAN MANIFOLDS

## Physical interpretation

- Geodesic  $g(t)$  from  $p$  to  $q$  is local minimizer (stationary point) of energy  $\int |g'(t)|^2$  by comparison with close neighbor paths joining  $p$  and  $q$ .

# LAPLACE

- Use computations in the context of Laplace operator or its generalizations and modifications to analyze and classify geometry of
  - shapes
  - images

## **Reason:**

Laplace operator and its eigenfunctions and eigenvalues are related to shape of geometric objects.

## LAPLACE

**Wave equation:**  $u_{tt}(t, x) = \Delta_x u(t, x)$

**Heat equation:**  $u_t(t, x) = \Delta_x u(t, x)$

**Separation of variables:**  $u(t, x) = v(x) \cdot T(t)$

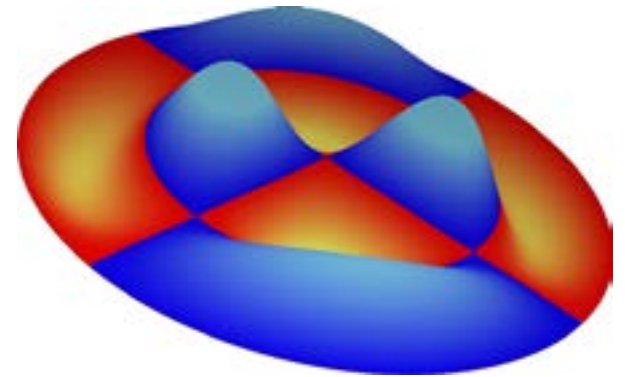
**Eigenvalue equation:**  $\Delta v = \lambda v$

**Eigenvalues (spectrum):**  $\lambda_1, \lambda_2, \dots$

**Eigenfunctions:**  $v_1, v_2, \dots$

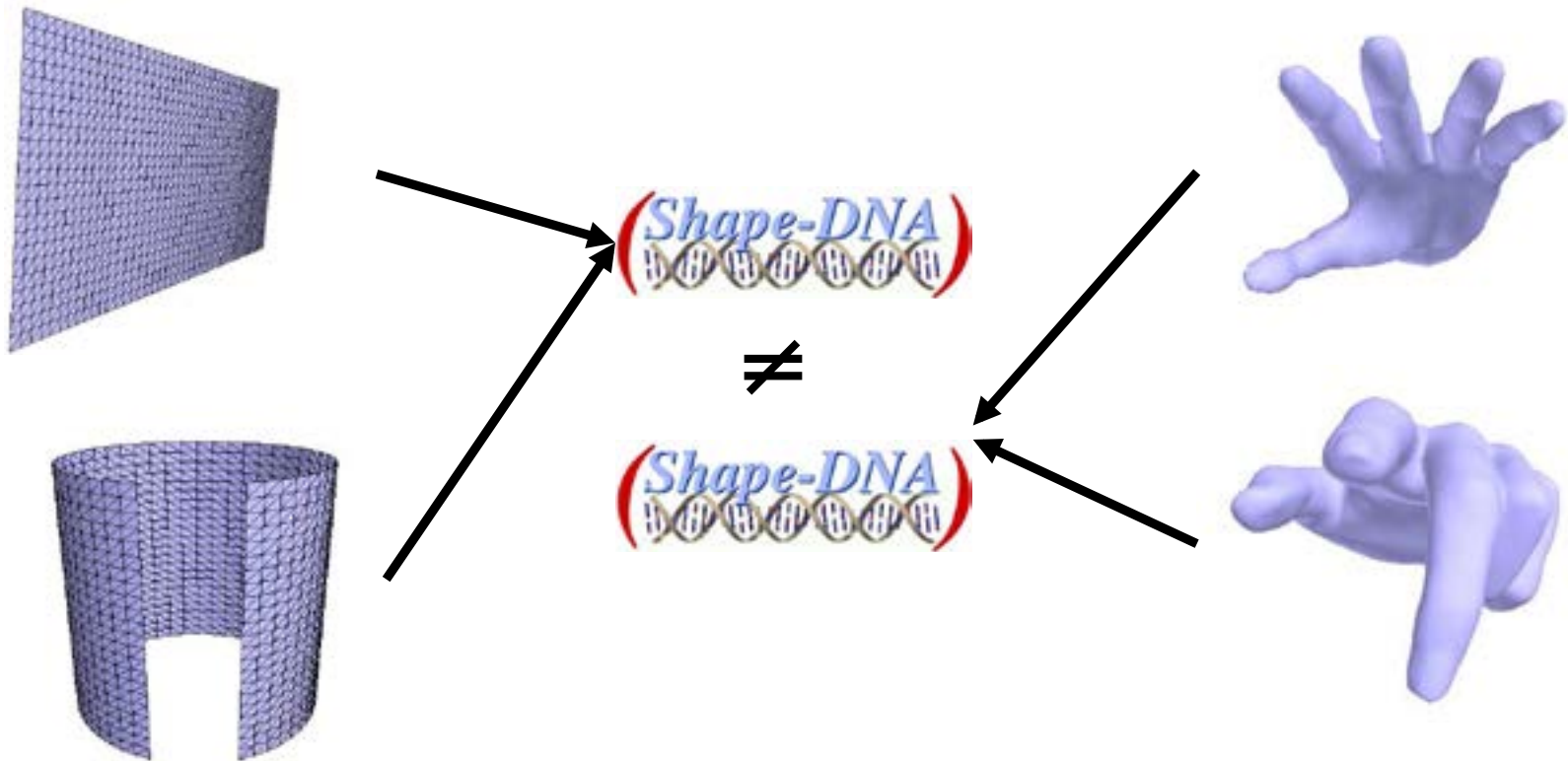
**Heat Kernel:**  $K(t, x, y) = \sum_{i=1}^{\infty} e^{-\lambda_i t} v_i(x) v_i(y)$

**Heat Trace:**  $\int_M K(t, x, y) = \sum_{i=1}^{\infty} e^{-\lambda_i t} \xrightarrow{t \rightarrow 0} A_S(c_0, c_1, c_2, t)$



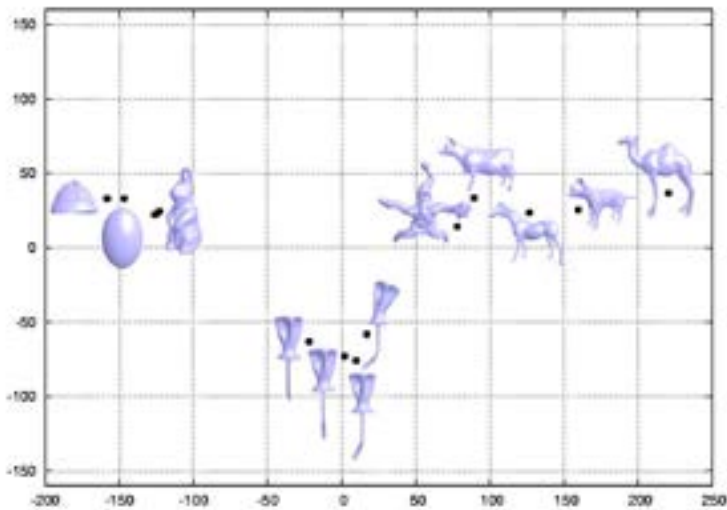
# LAPLACE SPECTRA AS SHAPE-DNA

- **Theorem:** Isometric shapes => Same Laplace spectra





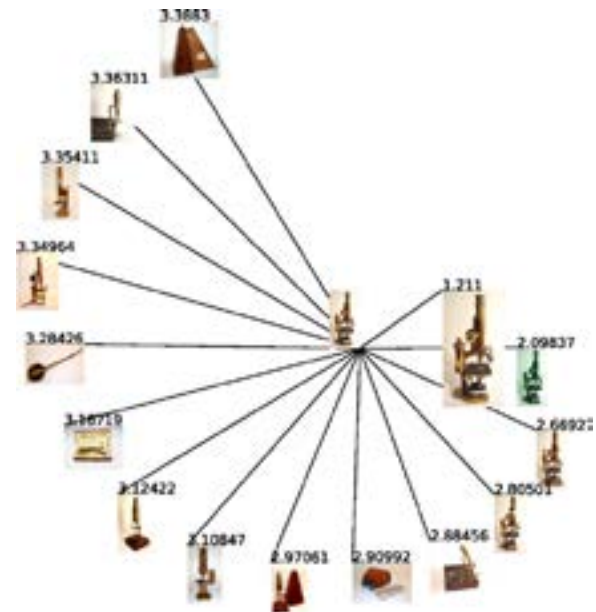
## SHAPE-DNA FOR GEOMETRY



### Applicable for:

- 2D surfaces
- 3D volumetric data

## SHAPE-DNA FOR IMAGES



### Applicable for:

- Monochromatic images
- Greyvalue images
- RGB images
- 3D (Voxel) images

# LAPLACE: SPECTRAL INFORMATION

## **Eigenvalues/Heat Trace determine:**

- Volume
- Volume of boundary
- Curvature integrals
- Euler characteristic
- ...

## **Spectrum can be used as „Shape-DNA“ for**

- Surfaces
- Solids
- Images
- ...

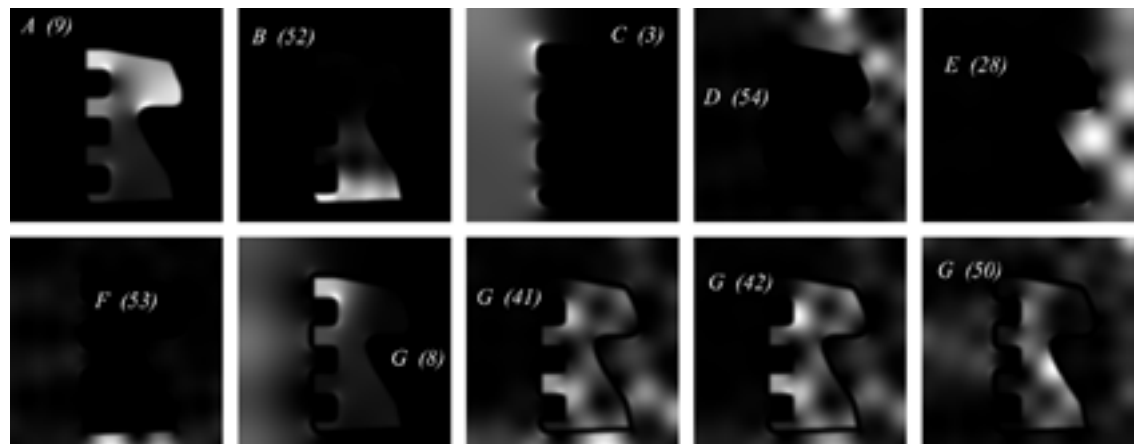
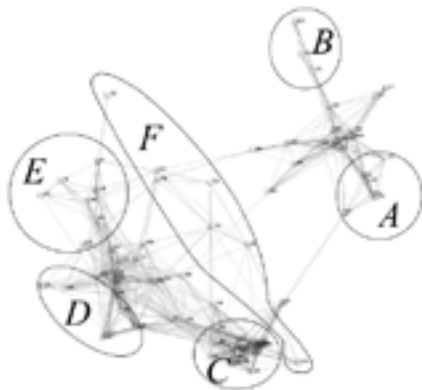
## GENERALIZATIONS AND CURRENT RESEARCH

- Laplacians on vectorbundles
  - Zero-sets of eigenfunctions
  - Hyperbolic manifolds



*Vais, A., Brandes, D., Thielhelm, H., Wolter, F.-E.: Laplacians on flat line bundles over 3-manifolds. Comput. Graph. 37, 718–729 (2013).*

- Extended descriptors (partial matching, ...)



*Berger, B., Vais, A., Wolter, F.-E.: Subimage sensitive eigenvalue spectra for image comparison. Vis. Comput. 1–17 (2014).*

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## **2. VIRTUAL REALITY**



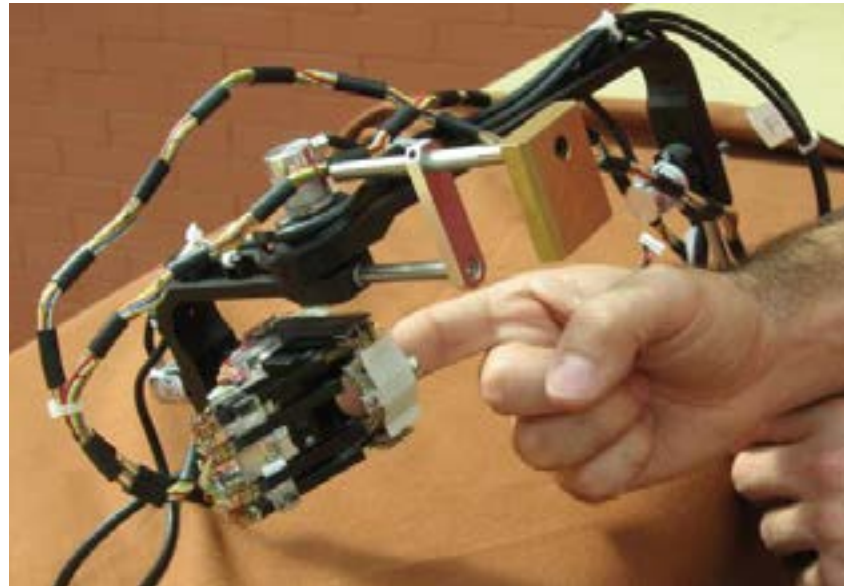
# VIRTUAL REALITY: BASICS

Three important areas for realistic Virtual Worlds

- A. Visual
- B. Tactile
- C. Haptic

# TACTILE SIMULATION

Physical Simulation Tactile Signal



- Physical Models yields relevant Tactile Data  $T(t)$
- Signal (usually) created by a Tactile array
- Relevant class of Signals to create illusion (Tactile Color)

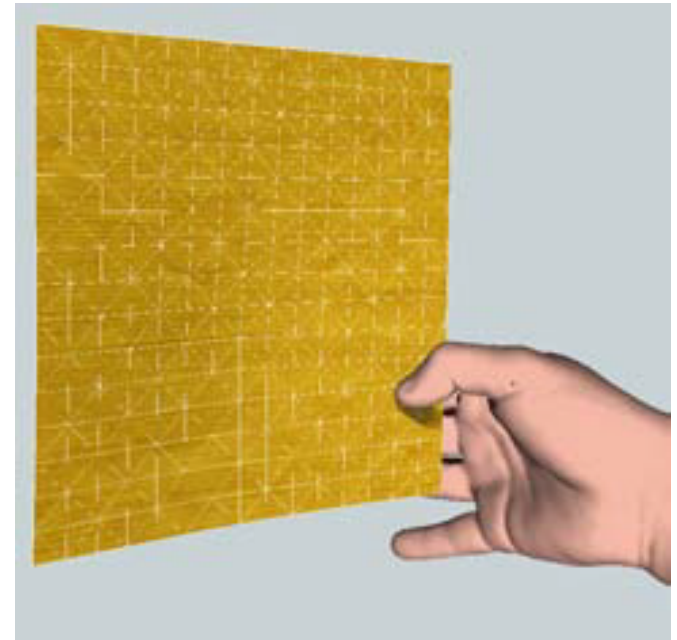
# HAPTIC SIMULATION

- Solve a Dynamical System

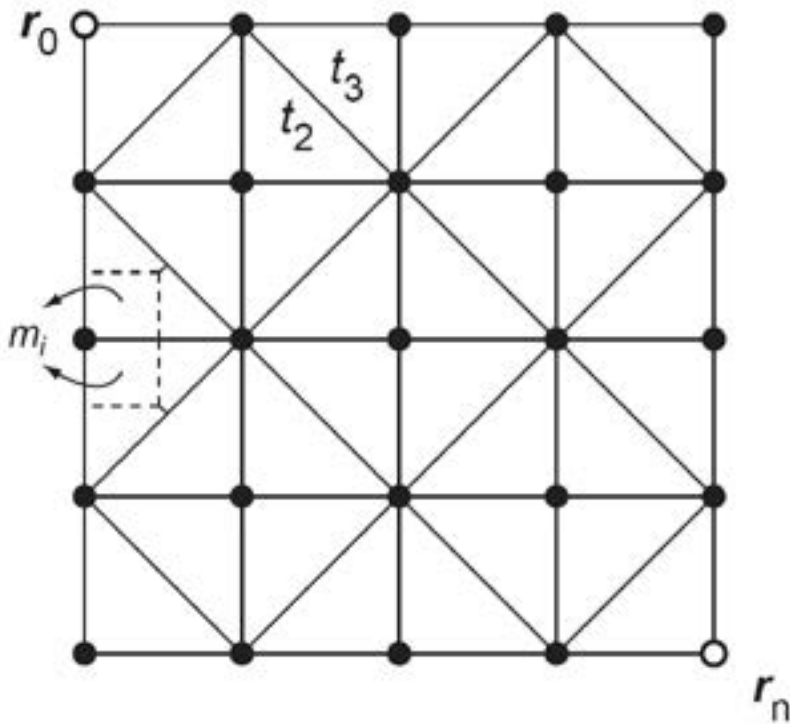
$$M \cdot x''(t) = F(x(t), x'(t))$$

$$x''(t) = M^{-1} \cdot F(x(t), x'(t))$$

- Appropriate Integration needed
- Usually fast methods are required to achieve 1000Hz



# HAPTIC SIMULATION



- Interpretion as a particle system
- Second order ODE:

$$r''(t) = F(r, r')M^{-1}$$

- Numerical integration of a  $6n \times 6n$  matrix



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## **3. PHYSICAL SIMULATION**

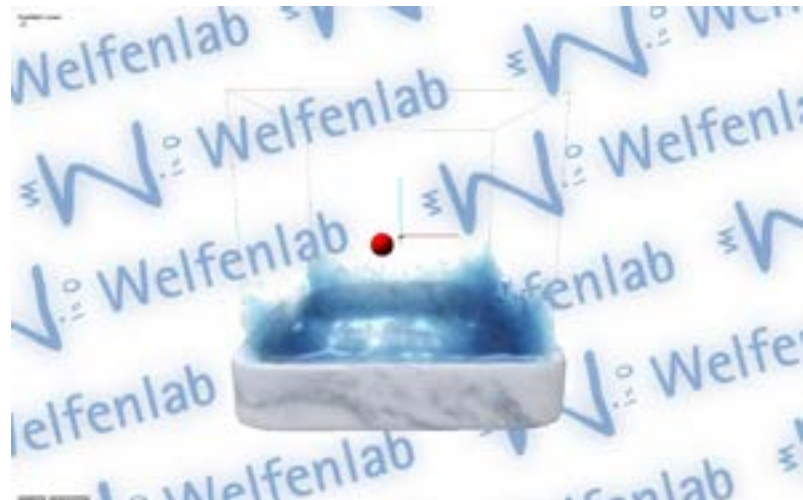


# GENERAL EXTENDABLE PHYSICAL SIMULATION FRAMEWORK



# COMPUTATIONAL METHODS FOR FLUID DYNAMICS

- Examination of different computational paradigms
  - Grid based (esp. Adaptive multigrid solvers)
  - Particle based (SPH, WCSPH, PCISPH, ISPH...)
- Research on fluid structure interaction (focussing on haptics)
- Relations to Riemannian Geometry as in  
*“Geometrical theory of Fluid Flows and dynamical systems”*,  
Tsutomu Kambe

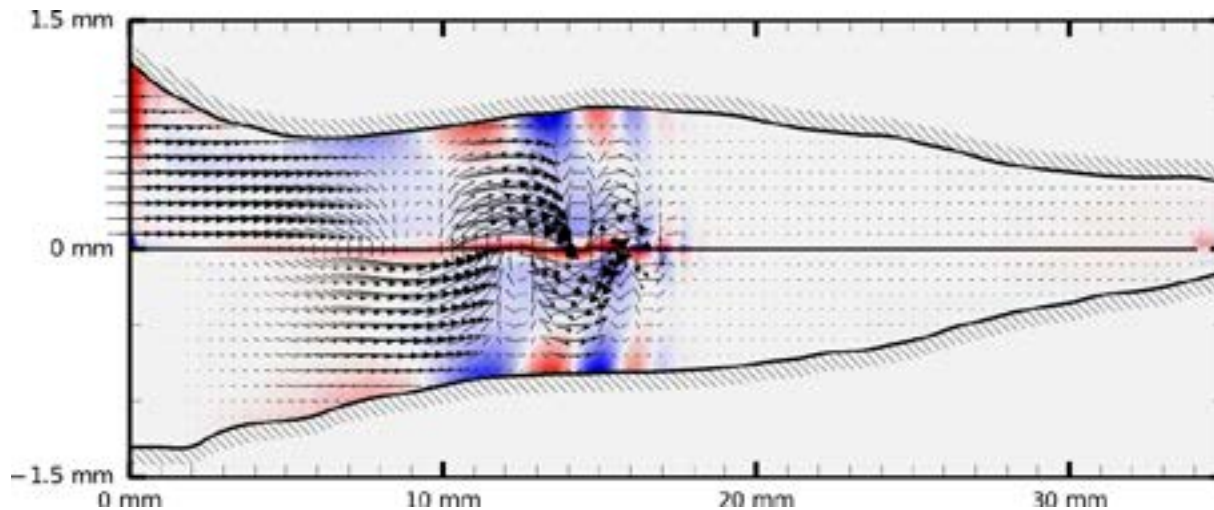


# HEARING MECHANICS

## 2D Simulation

- Accoustic streaming analysis
- FEM solution for cochlea mechanics in every single time step (involving Navier-Stokes, etc.)
- Solving a huge dynamical system

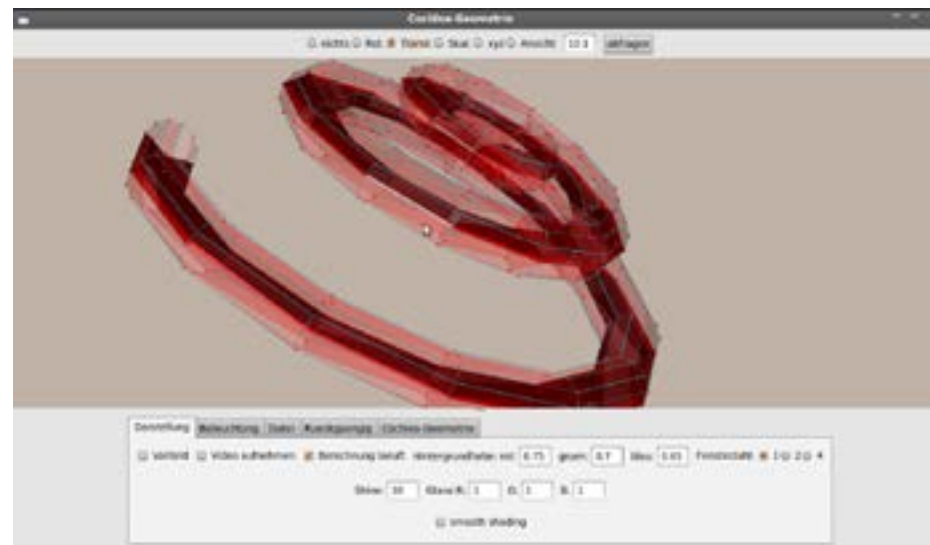
$$M(t) u''(t) + H(t) u'(t) + K(t) u(t) = F(t)$$



# HEARING MECHANICS

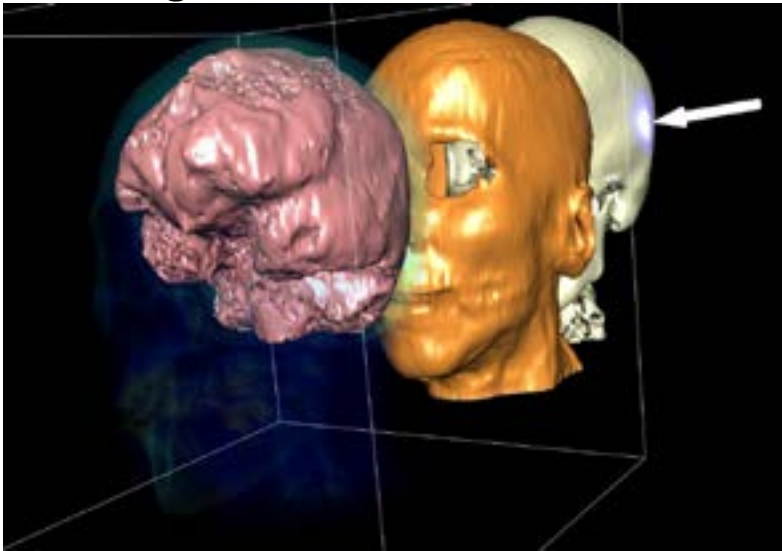
## 3D Simulation

- Extension to the 3D case
- Non-FEM approach (FDM / Lattice Boltzman)
- Use Cluster for calculations



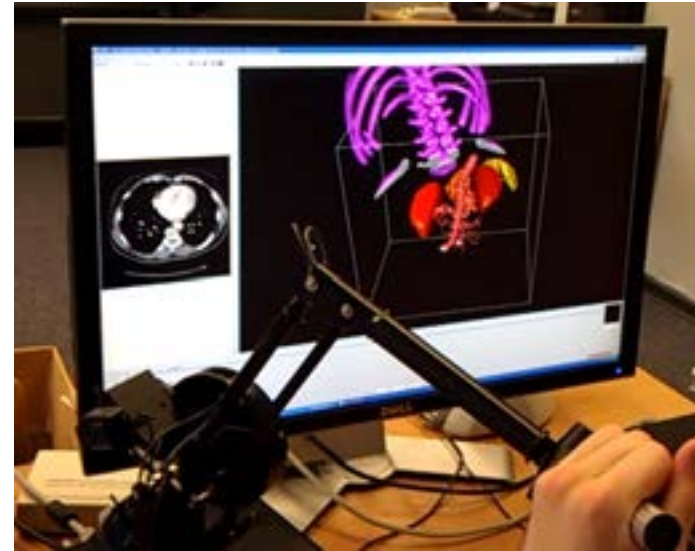
# HAPTIC INTERACTION WITH 3D VOXEL DATA

- Haptic Realtime Challenge
- Similar to “normal Haptic Simulation”
- Use raycasting to check for contacts
- Allow interactive deformations with haptic feedback
- Integrated into the YaDiV framework



*Vlasov, R., Friese, K.-I., Wolter, F.-E.: Haptic rendering of volume data with collision detection guarantee using path finding. Transactions on Computational Science XVIII. pp. 212–231. Springer (2013).*

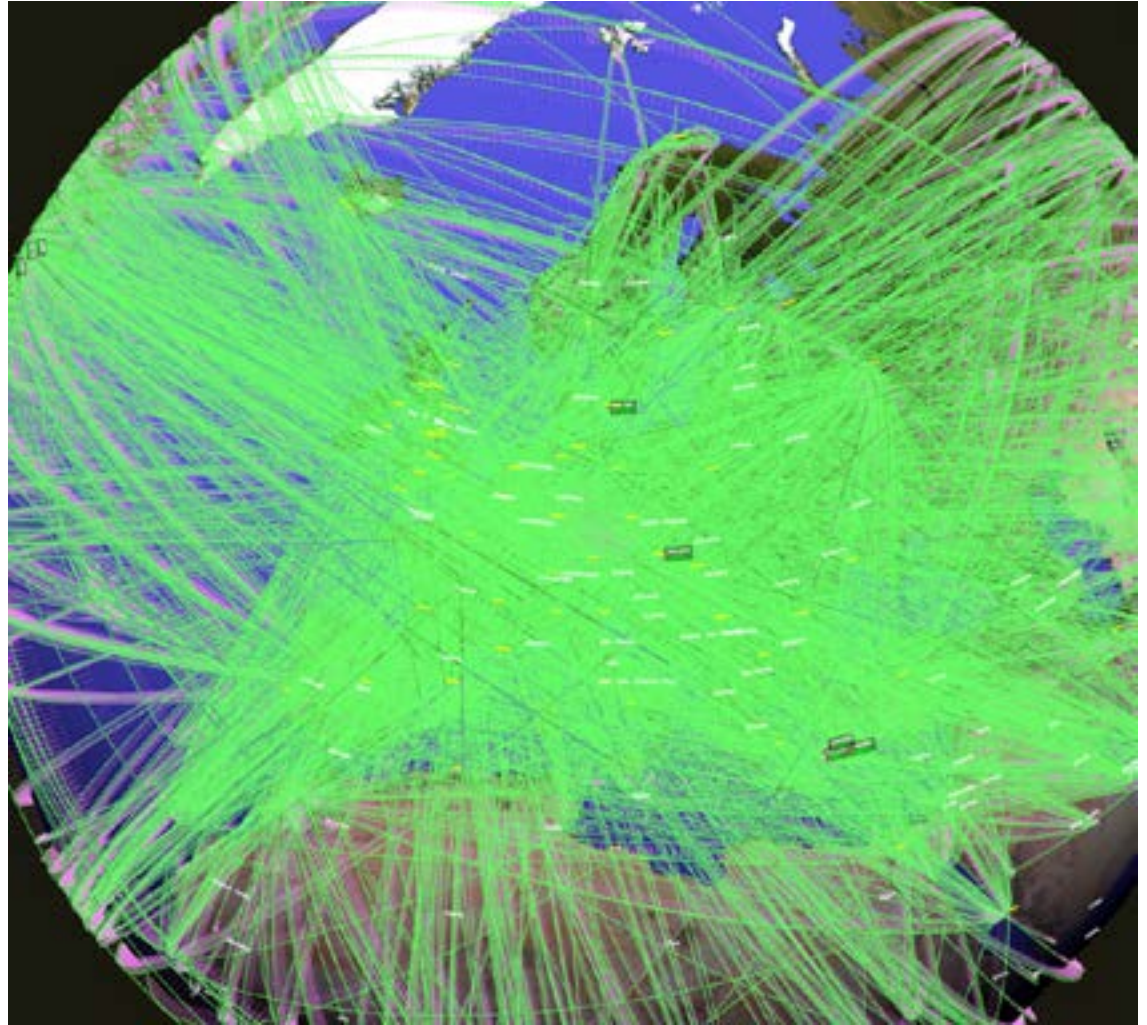
*Vlasov, R., Friese, K.-I., Wolter, F.-E.: Haptic rendering of volume data with collision determination guarantee using ray casting and implicit surface representation. Cyberworlds (CW), 2012 International Conference on. pp. 91–98. IEEE (2012).*



*Vlasov, R., Friese, K.-I., Wolter, F.-E.: Ray casting for collision detection in haptic rendering of volume data. Proceedings of the ACM SIGGRAPH Symposium on Interactive 3D Graphics and Games. pp. 215–215. ACM, New York, NY, USA (2012).*

# FLIGHT PATH SIMULATION

**Problem statement:**  
Many simultaneous  
flights



*Kuenz, A., Schwoch, G., Wolter, F.-E.: Individualism in global airspace-user-preferred trajectories in future ATM. Digital Avionics Systems Conference (DASC), 2013 IEEE/AIAA 32nd. pp. 1E1-1E13. IEEE (2013).*

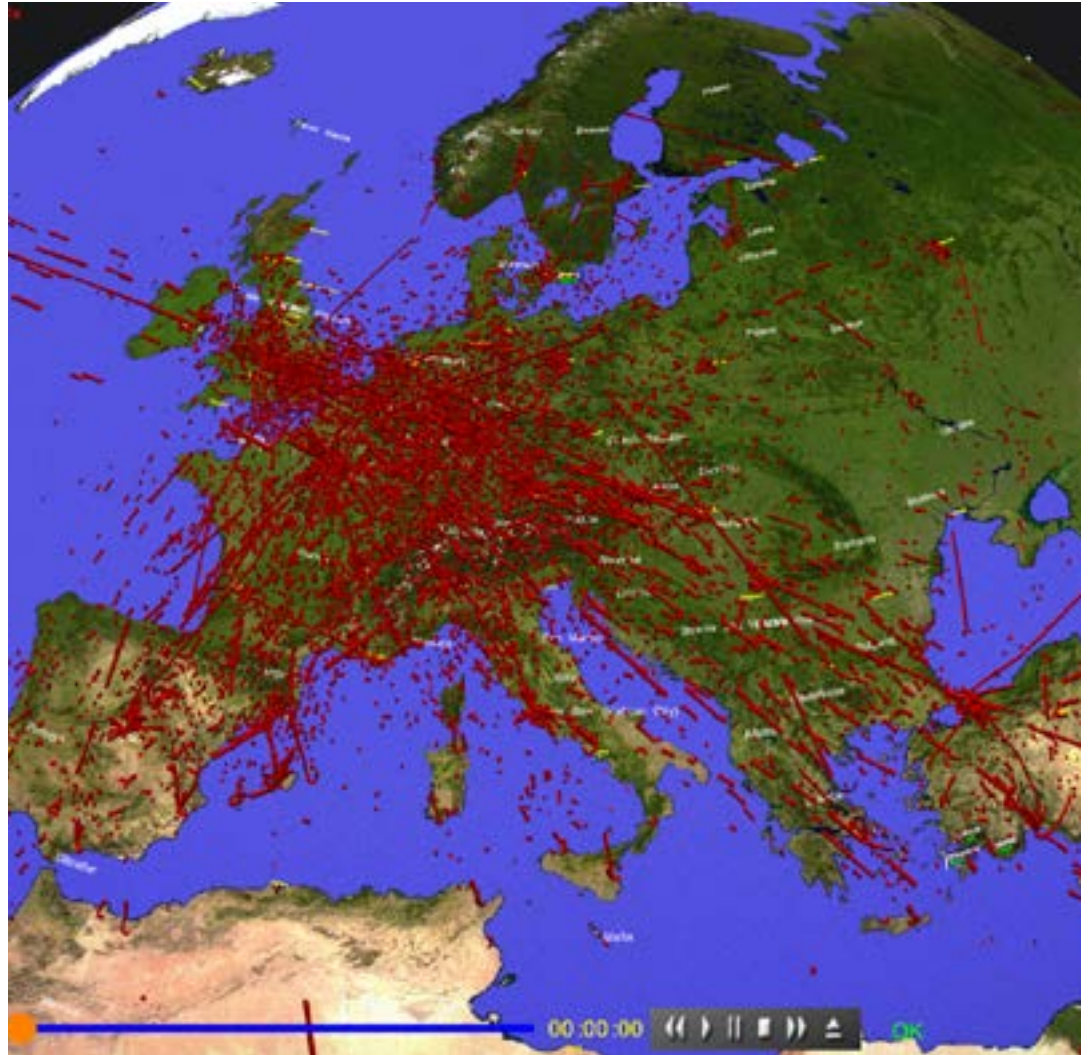
# FLIGHT PATH SIMULATION

## Problem statement:

Many simultaneous flights

Get a collision free dynamical system

- Use a 4 dimensional description (Hexadecimal Tree)
- Search adaptively in the 4 dimensional space





# FLIGHT PATH SIMULATION

## Problem statement:

Many simultaneous flights

Get a collision free dynamical system

Consider delaying flights to avoid collisions

The system has to be very fast to be of use



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## 4. VISUALIZATION



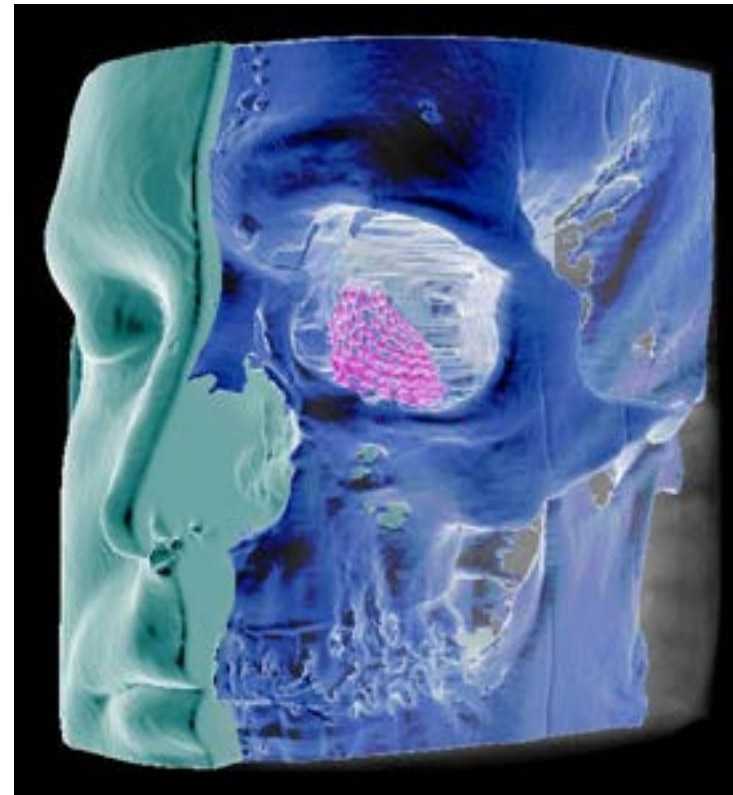
# VISUALIZATION BASED ON FRACTAL MODELS

- Discrete dynamic system generated by a self similar map  
 $\lim(S^n(I))$   
with  $I$  being a initial seed set.
- Limit set  $\rightarrow$  fractal
- Used to generate visualization data



# 3D VOXEL BASED-VOLUME VISUALIZATION

- A function  $P(x)$  defined on 3D Voxel data
- Visualization of intensity data of this function (YaDiV)

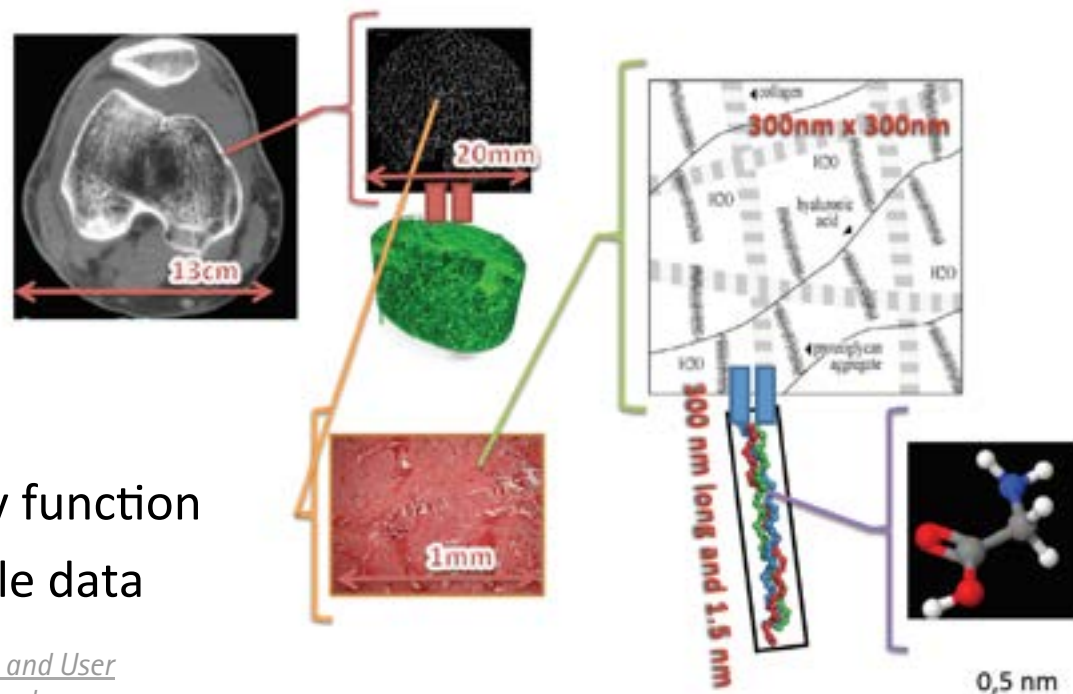


# BIOMED. MULTI SCALE VISUALIZATION

- Some of the 3D biomedical data can be visualized with YaDiV
- Important to handle processes that change over time on different time and length scales
- Visualized data
  - Computational results
  - Appropriate physical models involved

## Example

- Biomedical CT-Data
  - Radon transform density function
  - Segmentation visualizable data



# Computational Geometry + Topology

